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# Volatility Patterns of Islamic Equity Funds: Using Hybrid Machine Learning and GARCH Models

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## Abstract

This paper examined the volatility of Islamic equity funds using daily price data from 2009 to 2023. Volatility models for S-GARCH, GJR-GARCH, E-GARCH, SVM-GARCH hybrid, and neural network are implemented to measure the accuracy of volatility prediction. The results of this study show that past volatility, unconditional variance, and lagged conditional variance are revealed as strong predictors of Islamic funds volatility. In light of the findings, the squared residuals lagged conditional variance, and constant terms show a statistically significant positive effect on the ability to predict the volatility of the Islamic funds using the various models. Furthermore, employing historical information on volatility and the features of the specific market conditions vastly boosts the accuracy of the volatility forecast for the KMI30 index. This research demonstrates that SVM-GARCH hybrid models with linear kernel and neural network model offer high accuracy in Islamic funds volatility forecasting, as indicated by their corresponding root mean square and absolute error. Such implications benefit policymakers and practitioners in the Islamic financial market when policy making uses volatility models. These implications might be applied to risk management, economic stability, and market regulations. Additionally, regarding portfolio investment or financial market decisions, the SVM-GARCH hybrid and neural network model could be utilized in risk management, risk performance, and decision-making. Thus, this study will serve as a foundation for decision-making within the Islamic market.

**Keywords:** Islamic funds, Volatility prediction, GARCH models, SVM-GARCH models, and Neural networks.

## 1. Introduction

The volatility of stock markets is the subject of numerous empirical studies. More specifically, this got attention after various financial crises. The core goals of the study are to analyze the dynamics of mutual funds volatility and identify the driving factors determining the volatility of these markets. This goal was approached by employing distinct methodologies considering specific and crucial financial time series characteristics: asymmetry, volatility clustering, extended

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memory, and fat tails. This set of observed patterns allows researchers to gain support for enhancing volatility modelling, managing portfolios, allocating capital effectively, and choosing financial assets.

Volatility plays a role in finance, indicating the risk associated with assets (Dhingra et al., 2024). It quantifies the market uncertainty impacting risk management, investment strategies, and economic policies. It is equally important to understand how changes in a time series relate to volatility. Models developed for this purpose must establish the appropriate stochastic process (Guyon & Lekeufack, 2023). Understanding how different variables interact over time must also be captured. Analysis through econometric tools such as Bollerslev (1986)'s Generalized Autoregressive Conditional Heteroskedasticity (GARCH) based on current conditional volatilities helps properly evaluate variances.

Such models as Autoregressive Conditional Heteroskedasticity (ARCH) models and other ARCH-based stochastic volatility models are also widely used in forecasting volatility for finance and economics-related time series data. As an illustrative case, Choudhry et al. (2016) analyzed how business cycles correlated with market volatility experienced in the United Kingdom, the United States, Japan, and Canada. In this case, four business components were associated with stock market volatility. Choudhry et al. (2016) understood the effect of the crisis on any one of the components. Forecasts were also elaborated, and market volatility forecasted business cycles can be provided over short-term periods of activity and downturn in four countries.

Aggarwal et al. (1999) utilized the GARCH models to explore fluctuations in the condition volatility of developing stock indices. Their results indicate that these markets have undergone shifts toward heightened volatility. Moreover, they established a link between volatility changes and country-specific political and global economic occurrences. In a research conducted by Al-Khouri and Abdallah (2012), they analyzed the behaviour of developing markets considering financial crises using the family of GARCH models. The authors found an inverse correlation between stock returns and volatility, suggesting that greater volatility is associated with returns. Furthermore, they concluded that the volatility of developing markets expressively amplified during times of financial instability.

In another related study, Singhania and Anchalia (2013) investigated the reaction to the global financial crisis on Asian stock markets. Their research emphasized the importance of comprehending market volatility for policymakers in making informed decisions during economic downturns by applying an Exponential Generalized Autoregressive Conditional Heteroskedasticity (E-GARCH) model to Asian markets (China, Hong Kong, India, and Japan) during the crisis of Europe and the subprime crisis to volatility behaviour. The authors found positively significant relationships between the volatility and the subprime crisis of the examined stock markets. In the context of the Qatar exchange, Al-Khouri and Abdallah (2012) used the E-GARCH model along with a dummy variable to assess the relationship between volatility and the liberalization of the stock market. They found no significant changes after removing restrictions on foreigners' participation in the stock market's conditional variance. However, the findings suggested that volatility became more persistent following the financial liberalization of the market.

Moreover, the study by Vu (2015) used data from 27 Organisation for Economic Co-operation and Development (OECD) countries to inquire whether market volatility influenced the growth trend of the output. Consequently, the trend in stock market played an important role in presenting a clearer understanding of OECD nations. Fang et al. (2018) studied the impact of News-Implied Volatility (NVIX) on market volatility within established countries through GARCH models. The predictability increased since the features and mechanism of nature explained a positive relationship between the G7 market variance and that of NVIX. As such, they advised that investors and other professionals can use the US economic factor as a litmus paper to predict market volatility trends among advanced countries. Similarly, Guesmi et al. (2013) studied the pattern of volatility in the OECD markets with a Markov-Switching-GARCH model for the period 2004-2010, discovering that volatility was very frequently connected and synchronized with the financial downturn, for example, the global economic crisis. In the literature, such a connection between volatility and market spillovers is also considered by other researchers, such as Lien et al. (2018) and MacDonald et al. (2018)

Researchers' interest in analyzing stock market volatility continues to increase—the importance of the volatility of the advanced market evidences this. In particular, Nasr et al. (2016) and Burhanuddin (2020) used various GARCH models to investigate the volatility of the market index. Shahzad et al. (2017) examined the dependence of traditional stock markets on the volatility of the US, UK, and Japan from 1996 to 2016. According to their study, volatility can strongly influence the behaviour of markets. The effect of volatility on the spillover of the impact between Asian stock markets was also investigated by Majdoub and Sassi (2017) .

Arfaoui and Ben Rejeb (2021), taking Islamic indices, deeply investigated the volatility trends of markets on various financial crises and tested how the crises affected the markets driven by the abovementioned risk factors. Their research based on the regression and GARCH models concluded that Islamic indices respond to the factors during financial crises and economic instability. Nasr et al. (2014) demonstrated that it is possible to provide new information on returns within the equity market through changes in volatility patterns along with long-term memory models. Moreover, Chiadmi and Ghaiti (2014) employed the GARCH to examine and compare volatility patterns between stock and conventional indices.

According to the results of the various studies, Islamic stock markets responded to the crisis. Moreover, Islamic stock markets showed strengthened volatility compared to interconnected markets, which reflects the perfect resistance of Islamic instruments from the global financial and economic downturn, which affected the real economy. Mezghani and Boujelbène (2018) studied the volatility differences between Islamic stock markets and the oil markets based in the countries of the Gulf Cooperation Council. They found that international investors also impact the volatility of these countries. When an international investor anticipates instability and volatility in these markets, they withdraw their investments from these markets, which affect the Gross Domestic Product (GDP) of these countries. So, volatility forecasting plays a vital role within the market for risk management, decision analysis, option evaluation, and security appraisal Charles (2010) and Poon and Granger (2003).

Mandelbrot (1967) indicated that in time series data, the tails are associated with kurtosis, and non-normal probability distribution exhibits more potential for volatility because of patterns. Financial time series very often display such patterns and are characterized by variations of so-called volatility clustering. This specific kurtosis and the volatility clustering can be analyzed using GARCH and ARCH models. The GARCH family under discussion also includes various model extensions and modifications, the E-GARCH model projected by Nelson (1991). All of these have high precision when analyzing short-term variations. Thus, they rely on data and possess datasets and can provide better insight into short-term fluctuations than long-term ones, which are the disadvantages of these models.

Encalada-Dávila et al. (2021) used artificial neural network (ANN) to respond to these limitations. In recent years, such models have succeeded in economic and financial applications. They have proven to be effective in different scenarios. For example, Ormoneit and Neuneier (1996) used density networks and multilayer perceptron to predict volatility in the Deutscher Aktienindex (DAX) market . Similar studies used ANN to examine the implied volatility of the Standard & Poor's (S&P) 500 Index and IBEX 35 options, respectively Gonzalez Miranda and Burgess (1997) and Hamid and Iqbal (2004). Barunik et al. (2016) proposed applying ANN when predicting energy market volatility, where they also had a good prediction performance. Oliveira et al. (2017) developed a system for forecasting returns, volatility, and trading volume based on microblogging data with mood and attention indicators. Maknickienė and Maknickas (2012) used Long Short-Term Memory (LSTM) networks to predict currency transactions and exchange rates, while Chen et al. (2015) predicted stock market returns using the LSTM model. Thus, the examples above confirm the relevance and effectiveness of ANN in financial and economic forecasting.

Various studies confirmed that fusing machine learning techniques with models works better in terms of results than using only models in multiple unfamiliar situations. For example, Chen et al. (2015) combined feedforward networks with an E-GARCH model for the investigation of price volatility of Taiwan stock options and found better results. Two models were developed later by Hajizadeh et al. (2012), which fused feedforward networks with E-GARCH for volatility forecasting. More research using a network model was conducted by Kristjanpoller et al. (2014), who investigated a network model for predicting volatility in three Latin American markets. Another successful use of a network model was described by Luo et al. (2017), who used an ANN model to predict stochastic volatility, thereby demonstrating that the combination of machine learning techniques with models has achieved good results.

A GARCH-SVM (Support Vector Machine) hybrid model for analyzing volatility patterns in Islamic finance is valuable for a few key reasons. First, Islamic equity funds can exhibit complex, non-linear relationships in their volatility behaviour. The GARCH model alone may not be sufficient to capture all the non-linearities present in the data. Integrating a machine learning technique like SVM can help the model identify and adapt to the non-linear patterns in Islamic fund volatility.

Second, the hybrid GARCH-SVM model can leverage the strengths of both the parametric GARCH model and the nonparametric SVM approach. This combination can lead to more accurate and reliable volatility forecasts than using either model in isolation. Improved volatility forecasting is crucial for Islamic finance risk management, asset allocation, and investment decision-making. Third, Islamic finance operates under specific principles and guidelines, which can influence the volatility dynamics of Islamic equity funds. A GARCH-SVM hybrid model can be tailored to capture the unique features and complexities inherent in Islamic financial instruments and markets. This customized approach may outperform conventional volatility models in Islamic finance.

Last, the hybrid model combines the parametric strengths of GARCH with the non-parametric capabilities of SVM. This integration can make the volatility modeling more robust to various market conditions and data characteristics encountered in Islamic finance. The flexibility of the hybrid approach allows for better adaptation to the evolving nature of Islamic equity fund markets. By leveraging the complementary strengths of GARCH and SVM, the hybrid model can provide a more comprehensive and accurate representation of the volatility patterns in Islamic equity funds. This can lead to valuable insights for risk management, investment strategies, and the overall understanding of Islamic finance dynamics.

In this study, we use the GARCH family, neural network (NN), SVM, and hybrid models to envisage volatility in Islamic equity funds time series data as part of our research. Our methodology employs GARCH estimation methods combined with the SVM and NN approach. Our strategy was performed by applying the system that determines the best window size for predicting volatility. Specifically, we use the best window size for the first time to investigate the volatility patterns in Islamic funds. We are interested in finding out why this method was employed, learning more about the characteristics of the Islamic stock market, and understanding why it is so hard to predict.

The rest of this article is organized as follows. Section 2 outlines the machine learning and statistical methodologies employed to estimate volatility. Section 3 analyses the results of examining Islamic funds' daily net asset value. Finally, in Section 4, conclusions and suggestions for future research are drawn from this review.

## 2. Material and methodology

The GARCH model is a statistical technique used to analyze and forecast a financial time series's volatility (or fluctuations), such as stock prices or fund returns. It works by looking at past volatility patterns to predict future volatility. The model assumes that the volatility of a financial asset is not constant over time but varies based on previous volatility levels. GARCH models are handy for capturing the clustering of volatility, where periods of high volatility tend to be followed by periods of high volatility and vice versa.

On the other hand, SVM is a machine-learning algorithm that can be used to classify and perform regression tasks. In the context of your research, the SVM model is used as a nonparametric approach to complement the GARCH model in capturing the volatility patterns of Islamic equity funds. The SVM model can identify complex, non-linear relationships in the data that may not be fully captured by the GARCH model alone. By combining the GARCH and SVM models, the hybrid approach can take advantage of the strengths of both techniques to provide a more comprehensive and accurate analysis of the volatility patterns.

The GARCH-SVM hybrid model integrates the GARCH and SVM models to create a more powerful tool for analyzing and forecasting the volatility of Islamic equity funds. The GARCH model captures the linear, time-varying aspects of volatility. In contrast, the SVM model identifies and accounts for the non-linear, complex patterns in the data. The hybrid approach allows the model to leverage the complementary strengths of the GARCH and SVM techniques, leading to improved performance and a better understanding of the underlying volatility dynamics.

Volatility is a relatively easy-to-understand but arduous term to define precisely. In the early days, even before Markowitz (1952) work on Portfolio Theory, the concept of volatility was unclear. However, Markowitz (1952) pioneering approach provided volatility with a specific definition: the standard deviation. This approach allowed for the integration of mathematics into finance in a new way and has led to a fundamental change in the discipline. Importantly, empirical volatility, the extent of the fluctuation of the value of financial assets, is directly connected to uncertainty, a fundamental tenet of all economic models. As the situation with the interconnected financial markets has developed, so too have rather long periods of uncertainty, which make the significance of volatility more pronounced.

Volatility is the risk proxy critical in many areas – from risk management to asset pricing. The omnipresence and omnipotence of volatility as a concept require it to be included in all models. An excellent example of the importance of one of the risk measures, volatility, is the Basel Accord, established in 1996. This international regulatory document instituted a pillar emphasizing volatility as a measure of risk. According to Karasan and Gaygısız (2020), the Basel Accord highlights the importance of systemic stability in maintaining financial functions.

Following the seminal works of Black (1976), Andersen and Bollerslev (1997), Dokuchaev (2014), and De Stefani et al. (2017), a vast body of research has been growing based on estimating volatility. This tradition is mainly built on using ARCH and GARCH-type models to predict volatility. Unfortunately, they are associated with some disadvantages, namely, volatility clustering and lack of information, leading to a fall in the models' effectiveness. Numerous models have been introduced to fix the problem, but current fluctuations in financial markets and advancements in machine learning have forced scholars to rethink volatility estimation. This changed reality has made researchers reconsider the current outdated approaches and take steps to integrate machine learning into the process of volatility estimation.

We aim to test which models based on machine learning will improve the prediction performance. We will implement different machine learning algorithms, such as support vector regression, neural networks, and deep learning. This will allow a comparison of how good the formulated prediction is and how good it can be under optimal formulations. Thus, volatility modelling measures and considers uncertainty, allowing us to model reality. Therefore, the return volatility – the length of the difference between the model and the reality, is an important measure. Reality is measured as realized volatility, calculated from realized variance, and calculated as the sum of the squares of the realized returns. Realized volatility is calculated as the square root of the realized variance. Measuring the efficiency of volatility forecasting methods means calculating the return volatility.

$$r_t = \log\left(\frac{p_t}{p_t - 1}\right) \text{ and } \sigma = \sqrt{\frac{1}{n-1}\sum_{n-1}^{N}(r_n - \mu)^2}. \tag{1}$$

In Eq. (1), $p_t$ represents the daily price of the fund, $r$ represents the return, $\mu$ denotes the mean of returns, and $n$ signifies the total number of observations.

The volatility estimation plays a crucial role in the reliability and accuracy of many analyses. Thus, this paper discusses the degree of difference between the classical and machine learning-assisted methods of forecasting volatility. The goal is to outline the improved power of machine learning-assisted estimation methods. To conduct further comparisons between the classical and machine-learning-assisted forecasting methods, this paper begins with generating classical volatility models. The classical volatility models vary and may include, among others, ARCH, SGARCH, EGARCH, GJR-GARCH, etc.

### 2.1. ARCH model

The ARCH model Engle (1982), is a univariate model that relies on historical asset returns for estimation. The ARCH (p) is formulated as follows in Eq. (2).

$$\sigma_t^2 = \omega + \sum_{k=1}^{p}\sigma_k(r_t - k)^2, \tag{2}$$

where, $r_t = \sigma_t\ \epsilon_t$ and $\omega > 0$ and $\sigma_k \geq 0$. In this parametric model, where $\epsilon_t$ is assumed to be normally distributed, certain assumptions need to be met to ensure strictly positive variance.

ARCH is a non-linear and univariate model from all the equations mentioned above. Therefore, we can define it as estimating volatility using the squared values of past returns. Differently, ARCH has the unique characteristic of capturing time-varying conditional variance. According to this fact, «volatility changes over time: significant changes are followed by large changes of either sign; more insignificant changes follow insignificant changes Mandelbrot (1963). Due to this characteristic, it can have significant announcements if the market has major announcements.

### 2.2. SGARCH model

However, despite the appealing features such as simplicity, non-linearity, ease of use, and easy adjustment for forecasting, the ARCH model has some shortcomings. It responds equally to both positive and negative shocks. Strong assumptions are imposed on such a parameter restriction. Slow adjustment to large movements can generate mispredictions. Hence, these drawbacks have made researchers seek extensions for the ARCH. In this regard, Bollerslev (1986) proposed the GARCH Model: Incorporating lagged conditional variance into the ARCH model will be the first-order improvement. The multivariate model is an autoregressive moving average model for conditional variance involving p-lagged squared returns and q-lagged conditional variances. Regression analysis is thus improved only by including the p number of lagged conditional variances. Bollerslev (1986) proposed the GARCH to be as expressed in Eq. (3).

$$\sigma_t^2 = \omega + \sum_{k=1}^{q} \alpha_k r_{t-k}^2 + \sum_{k=1}^{p} \beta k \sigma_{t-k}^2. \tag{3}$$

To ensure the consistency of the GARCH model, it is necessary to estimate the parameters $\alpha$, $\beta$, $\omega$ and, where p and q represent the maximum lags in the model. Meeting the conditions $\omega > 0$, $\beta \geq 0$, $\alpha \geq 0$, and $\beta + \alpha < 1$ is essential for a consistent GARCH model.

The deficiencies of not capturing the impact of historical innovations characterize the ARCH model. However, the GARCH model rectifies this weakness by introducing measures of historical innovations, which are expressed in the previous equation due to the property of expressibility of GARCH models as infinite order ARCH. The following equation will illustrate how the GARCH model can be presented as an infinite order of the ARCH.

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2. \tag{4}$$

Replacing $\sigma_{t-1} t^2$ in Eq. (4) by $\omega +\ \alpha\ r_{t-2}^2 +\ \beta\ \sigma_{t-2}^2$

$$\begin{aligned} \sigma_t^2 &= \omega + \alpha r_{t-1}^2 + \beta(\omega + \alpha r_{t-2}^2 + \beta \sigma_{t-2}^2), \\ \sigma_t^2 &= \omega(1+\beta) + \alpha r_{t-1}^2 + \alpha\beta\sigma_{t-2}^2 + \beta^2 \sigma_{t-2}^2. \end{aligned} \tag{5}$$

Similarly,

$$\sigma_t^2 = (1 + \beta + \beta^2 + ...) + \alpha \sum_{k=1}^{\infty} \beta^{k-1} r_{t-k}. \tag{6}$$

There are multiple reasons why GARCH is used in volatility modelling. Firstly, the returns fit the GARCH model well, mainly due to volatility clustering. Secondly, as the GARCH does not consider the returns to be independently distributed, the GARCH models the leptokurtic nature of the returns, which is an empirical property of the returns. Despite the robust framework and instant appeal to the shocks, the weakness of the GARCH approach is its purely symmetric response to the shocks, as determined in the research of Karasan and Gaygısız (2020). Glosten et al. (1993) suggested a solution to symmetric shock responses called the GJR-GARCH model.

### 2.3. GJR-GARCH model

It is convenient to take the GJR-GARCH model because the difference between the impact of positive and negative news is expressed asymmetrically: negative has much more of an impact than positive. In simpler words, concerning the asymmetry, the loss distribution is not offset but has a fat tail. Accordingly, the asymmetry can be introduced into the model as the parameter:

$$\sigma_t^2 = \omega + \sum_{k=1}^{q} \left( \alpha_k r_{t-k}^2 + \gamma r_{t-k}^2 I(\epsilon_{t-1} < 0) \right) + \sum_{k=1}^{p} \beta^{k-1} r_{t-k}. \tag{7}$$

In the equation, the parameter $\gamma$ makes the announcements asymmetric. If $\gamma = 0$, the answer to the previous shock is the same. $\gamma > 0$ implies that the former hostile shock response is above the last positive; if $\gamma < 0$, the answer to an earlier beneficial shock is above the negative one.

### 2.4. E-GARCH model

In addition to the GJR-GARCH model, an E-GARCH model was proposed to address the problem of considering the effect of asymmetric news. E-GARCH can be specified in logarithmic form, and there is no need to restrict keeping the volatility non-negative as the GARCH form so that it can describe the effect of asymmetry in more flexible ways:

$$\log(\sigma_t^2) = \omega + \sum_{k=1}^{p} \beta^k \log \sigma_{t-k}^2 + \sum_{k=1}^{q} \alpha_i \frac{|r_k - 1|}{\sqrt{\sigma_{t-k}^2}} + \sum_{k-1}^{q} \gamma_k \frac{r_{t-k}}{\sqrt{\sigma_{t-k}^2}}. \tag{8}$$

The crucial difference in this equation of the EGARCH model is the logarithmic transformation applied to the variance on the left side. This logarithmic transformation reflects the leverage effects: a mathematical association of past asset returns and volatility. Thus, when $\gamma < 0$, there is indeed a leverage effect, and when $\gamma \neq 0$, then there is no asymmetry in volatility.

### 2.5. SVM model

A distribution-free machine learning algorithm is a support vector machine. It acts according to the concept of structural risk minimization and is based on convex optimization Chia-Cheng et al. (2020). Additionally, support vector machines evaluate data and learn from samples as a technique for controlled learning utilized for regression analysis and classification. Vapnik (1998) was the first proponent of the support vector machine. SVM can classify non-distinguishable data sets as well as linearly distinguishable data sets. A unique feature of SVM is that it can be used in n-dimensional space. The n-dimensional data space is changed into another d measure of the data set with $d > n$ using a non-linear mapping. The same data can be classified into hyperplanes and suitable transformations.

SVM is a supervised learning algorithm used to solve both classification and regression problems. The main aim behind SVM is to obtain an optimal hyperplane capable of separating the two classes. Several lines can be split between the two classes. However, the specific line of interest is the one that separates the classes perfectly. In linear algebra, the best line is denoted as a hyperplane, which maximizes the distance between the classes' closest points. This distance between two points where the line separates is called the margin. The distance is the reason behind the name support vectors. As noted earlier, our main aim in using SVM is to maximize the margin of the support vectors.

Nonetheless, SVM is referred to as Support Vector Classification (SVC) when using it to solve a classification problem. Nevertheless, SVM can be used to solve the regression problem as well. In regression, the aim is to achieve a hyperplane with an optimal margin. Support Vector Regression (SVR) is referred to as the SVC variant.

In the context of GARCH modelling, we can combine these two models to create a hybrid approach known as SVR-GARCH. Let $x_t$ and $y_t$ be training datasets where $x_t \,\epsilon \mathbb{R}^p$ , $y_t \,\epsilon\, \mathbb{R}$ to demonstrate the theoretical underpinnings of SVR. Since the dataset is time-series structured, $x_t$ should be the $y_t$ lag quantities. Data is produced by a function that:

$$y_t = f(x_t) + \epsilon_t. \tag{9}$$

It is now necessary to define the decision function $f(x)$ in the following manner:

$$f(x_t) = w^T \phi(x_t) + b = \sum_{i=1}^{n} w_i \phi_i(x) + b, \tag{10}$$

where, $w = [w_1, w_2, \dots, w_n]$ and $\phi(x) = [\phi_1(x), \phi_2(x), \dots, \phi_n(x)]^T$ is a non-linear conversion to a space of higher dimension.

An $\epsilon$-insensitive loss function, $L_\epsilon(x, y, f(x))$, is proposed by Vapnik (1998) and is defined as follows.

$$L_\epsilon = \begin{cases} |y = f(x)| - \epsilon \\ 0, otherwise \end{cases} \tag{11}$$

According to Chung and Zhang (2017), errors below $\varepsilon$ are not penalized by the loss function. Error is disregarded in this instance, and no loss occurs. This suggests that data points inside or outside the vicinity are used to obtain f(x). We refer to this as $\rho$-insensitivity.

At this stage, the $\varepsilon$-insensitive loss is described by the slack variables $\varepsilon$ and $\varepsilon^*$, and the $\varepsilon$-SVR is given as:

$$\min_{w,b,\varepsilon,\varepsilon^*} \left[\frac{1}{2}||w||^2 + C\sum_{t=1}^{n}(\varepsilon+\varepsilon^*)\right], \tag{12}$$

subject to

$$y_t - w^{'}\phi(x_t) - b \leq \varepsilon + \varepsilon_t^t,$$

$$w^{'}\phi(x_t) + b - y_t - \leq \varepsilon + \varepsilon_t^t,$$

$$\varepsilon,\, \varepsilon_t^t \geq 0.$$

The second term in Eq. (12) refers to the $\varepsilon$-insensitive loss function and 1/2 $|(|w|)|^2$measures the function flatness. The Lagrangian method can be used to tackle this issue.

$$L_p = \frac{1}{2}||w||^2 + C\sum_{t=1}^{n}(\varepsilon+\varepsilon^2) - \sum_{t=1}^{n}\alpha_t(\varepsilon+\varepsilon_t - y_t + w^{'}\phi(x_t)+b) - \sum_{t=1}^{n}\mu_t + \varepsilon_t$$
$$- \sum_{t=1}^{n}\alpha_t(\varepsilon+\varepsilon_t - y_t + w^{'}\phi(x_t)+b) - \sum_{t=1}^{n}\mu_t^* + \varepsilon_t^*. \tag{13}$$

The following equations are made possible by the Karush-Kuhn-Tucker condition.

$$\frac{\partial dL_p}{\partial dw} = w - \sum_{t=1}^{n}(\alpha_t - \alpha_t^2)\phi(x_t) = 0,$$

$$\frac{\partial dL_p}{\partial db} = \sum_{t=1}^{n}(\alpha_t - \alpha_t^2) = 0,$$

$$\frac{\partial dL_p}{\partial d\varepsilon_t} = C - \alpha_t - \mu_t = 0,$$

$$\frac{\partial dL_p}{\partial d_t^2} = C - \alpha_t^2 - \mu_t^2 = 0,$$

where, the parameters should be $\alpha_t,\, \mu_t,\, \alpha_t^*,\, \mu_t^* \geq 0$. Furthermore, the SVR model's predicting performance greatly depends on the kernel function. Three categories of kernel functions exist, namely:

- Liner Kernal : $x_t x$
- Polynomial: $(x_t x + 1)^2$
- Gaussian: $e^{\frac{-||x-x_t||^2}{2\sigma^2}}$

## 2.6. SVR-GARCH model

SVM is a cutting-edge technique that has been discussed and is helpful in many different contexts. This study models the volatility using the GARCH model and SVM. The primary driving force behind this is the lack of a probability density function over returns in SVR Vapnik (1998). Researchers are given a powerful tool by the SVR-GARCH, which forecasts volatility and uses it to model risk.

Conventional risk models, like VaR, supply the model with standard deviation, which considers return volatility. Nevertheless, there aren't many additional tools available for predicting volatility than the conventional volatility model that can be used to estimate risk. The SVR-GARCH model yields more reliable results and a more appropriate strategy. The structure of the SVR-GARCH procedure is described below.

The equations below define the conditional mean estimation used to calculate the squared residuals, where $P_t$ represents the price and $r_t$ represents the return at time t. To obtain the squared residuals, the conditional mean estimation is employed, which is expressed as follows:

$$r_t = g(r_{t-1}) + a_t, \tag{14}$$

where, g is the mean equation estimation function determined by SVR, following Charles (2010), a volatility proxy is utilized because volatility is unobservable.

$$\sigma_t^2 = (r_t - \bar{r})^2. \tag{15}$$

Here, $\sigma_t^2$ refers to the conditional variance.

$$\sigma_t^2 = f(r_{t-1}^2, \sigma_{t-1}^2). \tag{16}$$

Here, $f$ represents the SVR decision function, while $\sigma_t^2$ denotes the conditional variance. The GARCH model assumes that the volatility of the financial time series (in this case, Islamic equity fund returns) is not constant over time but varies based on past volatility levels. This is a realistic assumption, as volatility in Islamic finance is often observed to be dynamic and clustering. GARCH models also assume that periods of high volatility tend to be followed by periods of high volatility and vice versa. This volatility clustering phenomenon is commonly observed in Islamic equity markets, where market shocks and uncertainties can lead to prolonged periods of heightened volatility. Further, the GARCH model assumes that the data's variance (or volatility) is not constant but depends on the variance's past values. This is particularly relevant for Islamic equity funds, as various Sharia-compliant factors and market events may influence their volatility.

The SVM model assumes that the relationship between the input variables (such as market factors) and the output variable (volatility) may be non-linear. This is an essential consideration for Islamic equity funds, as their volatility patterns can be influenced by complex, non-linear dynamics that linear models like GARCH may not fully capture. SVM models are designed to be flexible and adaptable to a wide range of data distributions and patterns. This aligns well with Islamic equity markets' unique characteristics and evolving nature, which may exhibit diverse and changing volatility behaviours over time. SVM models are generally robust to outliers and noise in the data, which can be a common challenge in Islamic finance, where market events and investor sentiments can introduce additional complexities.

The relevance of these assumptions to Islamic equity funds lies in the fact that the volatility patterns in these funds can be influenced by a combination of factors, including Sharia-compliant investment restrictions, investor behaviours, market dynamics, and regulatory changes. By incorporating the GARCH and SVM models, the hybrid approach can better capture the multifaceted nature of volatility in Islamic equity funds and provide more reliable and insightful analyses.

### 2.7. The neural network (NN)

The neural network, in turn, is the basis of deep learning. The neural network processes the data through several stages to help to decide. A neuron takes the dot product outcome as input and then processes it using an activation function to determine it. The variables are defined as follows in this example: the bias term as $b$ the weight as $w$ and the input data as $x$:

$$z = \omega_1 x_1 + \omega_2 x_2 + b. \tag{17}$$

The input data undergoes various mathematical operations within the hidden and output layers during this process. Typically, a neural network consists of three layers: the input, hidden layer(s), and output layers.

The input layer is responsible for the raw data. Meanwhile, proceeding from the input one to the hidden layer, people learn the coefficients accountable for concentrating on the relationships within the information. The number of hidden layers in the network can be different depending on its characteristics. In most cases, the hidden layer, situated between the input and output layers, makes non-linear transformations in terms of the activation function. The output layer produces the final output and helps in the decision-making process.

In Machine learning, we consider Gradient Descent a vital tool for minimizing the cost function. However, utilizing only Gradient Descent in neural networks isn't feasible due to their chain-like structure. Consequently, researchers introduced backpropagation to reduce the cost function efficiently. Backpropagation operates by calculating the observed and actual output error and propagating this error back to the hidden layers. This backward movement facilitates the adjustment of weights and biases across the network. The central equation in backpropagation takes the following form:

$$\delta^l = \frac{\delta_j}{\delta_x}. \tag{18}$$

In this context, we utilize $z$ to represent a linear transformation and $\delta$ to symbolize the error. Here, $z$ represents the linear transformation, while $\delta$ represents the error. Gradient Descent's optimization algorithm uses an update rule to search for the best parameter space ($w$, $b$) that minimizes the cost function. The update rule involves iteratively updating the parameters based on the gradient of the cost function concerning the parameters.

$$\theta_{t+1} = \theta_t - \frac{\delta_l}{\delta\theta_t}. \tag{19}$$

The gradient descent algorithm operates as follows: firstly, initial values are selected for the parameters (w and b); then, a step is taken in the direction opposite to the gradient, with the step size determined by the learning rate ($\lambda$); the parameters ($w$, $b$) are updated at each iteration based on the chosen step direction; and finally, the process continues until convergence is achieved, typically by monitoring the change in the cost function or the parameters. In summary, gradient descent starts with initial parameter values, takes steps toward steepest descent, updates the parameters, and iterates until convergence.

## 3. Empirical application

This study utilized daily return data from the Islamic equity funds in Pakistan. The data spanned from June 2009 to July 2023, covering the maximum available period. Since the COVID-19 pandemic did not significantly influence the Islamic equity funds in Pakistan, we did not analyze the impact of COVID-19 in our study. Using the available data on daily returns from the Islamic equity funds in Pakistan, we modelled return volatilities using four different approaches: GARCH, GJR-GARCH, E-GARCH, and SVR-GARCH. These models were applied to capture and analyze the volatility patterns in the return data. In the case of the SVR-GARCH application procedure, the best values for the parameters C, $\epsilon$, and $\gamma$ are determined using a grid search method, where they are considered for each value from 0 to 10. The best model is selected based on Akaike Information Criteria, which identifies the best way to build volatility of the chosen parameters. The Root Mean Squared Error and Mean Absolute Error are used to assess the performance of the found model. These indicators demonstrate the model's capability to capture volatility patterns using our approach. Before implementing these grid search and model evaluations, it is necessary to pre-process data and apply essential data transformations, such as difference or logarithm, if needed. As we see in Figure 1, the price changes over time are shown. As we see, the COVID pandemic did not significantly affect the Islamic index.

Return plots for the Islamic funds are depicted in Figure 2 below. It is evident from the plots that the returns fluctuate near zero around. This implies that Islamic funds, on average, are symmetrically distributed upwards and downwards and do not support any solid upward or downward trends in the long run. By inspection, one can understand how the returns move and identify places where they may be strongly and weakly correlated and where they are decidedly volatile.

Figure 3 illustrates Islamic equity fund returns' realized volatility from 2009 to 2023. As can be seen, several significant spikes correspond to the COVID-19 crisis. Although such data is associated with high volatility and uncertainty in the market conditions, it could hardly be considered substantial. While looking at such extremes, it is apparent that market participants were subject to several severe fluctuations and rapid changes concerning the Islamic equity funds. Nonetheless, drawing valid conclusions about the market based on the realized volatility without prior observation of the underlying dynamics is challenging. The reliability and accuracy of associated analyses largely depend on the volatility estimate. This section examines volatility estimations and predictions made using classical and

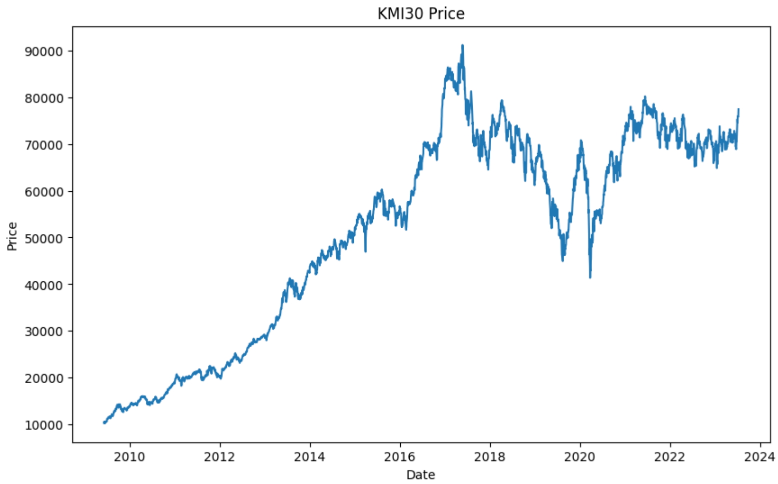


Figure 1: The net asset value of Islamic equity funds throughout 2009-2023.

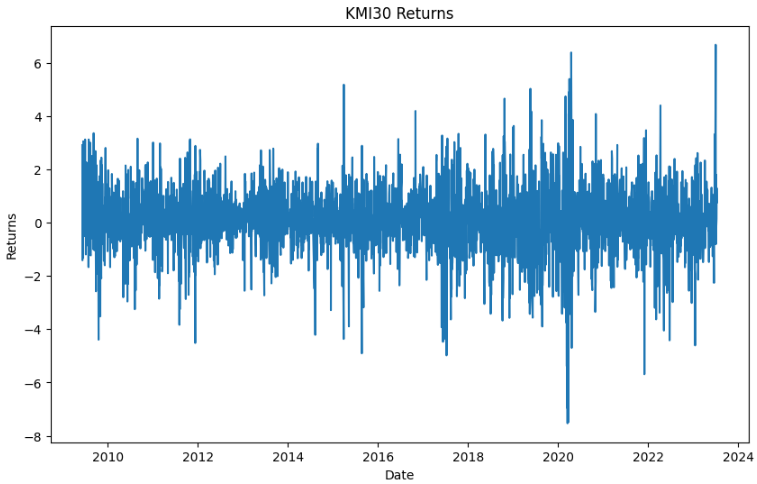


Figure 2: Funds return of Islamic equity funds.

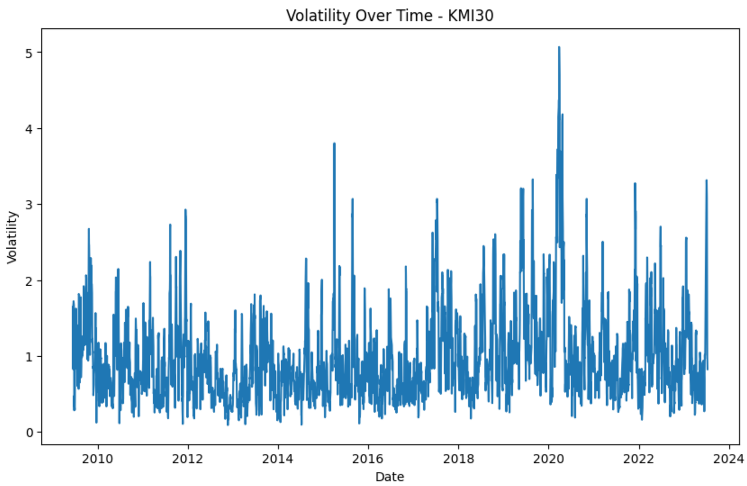


Figure 3: Realized volatility of the Islamic equity funds returns from 2009 to 2023.

ML-based techniques. This section aims to show the greater predictability of ML-based models. We first estimate the classical volatility models to compare these new ML-based models.

Table 1: Descriptive statistics of Islamic equity return and volatility.

| Statistics | Return | Volatility |
|---|---|---|
| Count | 3489 | 3489 |
| Minimum | -7.532 | 0.0902 |
| Maximum | 6.678 | 5.067 |
| Mean | 0.064 | 0.998 |
| Standard Deviation | 1.181 | 0.600 |
| Skewness | -0.249 | 1.712 |
| kurtosis | 3.703 | 4.509 |
| Jarque-Bera test $p-$value | 0.000 | 0.000 |

The vital descriptive statistics for Islamic equity fund's return and volatility are summarized in Table 1. The information revealed in Table 1 return variable had a mean of 0.064 and a standard deviation of 1.181, implying a moderately volatile return pattern. The volatility variable appeared positively skewed, reaching 1.171, and exhibited a kurtosis of 2.992, signaling a relatively peaked distribution. The Jarque-Bera test statistics and p-values of 0.000 are included for all variables, indicating that all three significantly diverge from a normal distribution. These statistics comprehensively summarize the Islamic equity fund return and volatility variables by stating their range, average, volatility, and distribution characteristics. Table 2 lists the unit root test results, including the test statistic of the Augmented

Table 2: Unit root test.

| | | t-Statistics | Probability |
|---|---|---|---|
| Augmented Dickey-Fuller test statistic | | -41.217 | 0.000 |
| Test critical values | 1% | -3.432 | |
| | 5% | 6.-2.862 | |
| | 10% | -2.567 | |

Dickey-Fuller test, which equals -41.217, and the corresponding probability – 0.000. This test helps determine whether a time series contains the unit root, which testifies to its non-stationarity. The feature of note concerns the probability p, which equals 0, revealing robust evidence against the unit root, thus making the time series stationary – low $p - value < 0.05$ in the two-tail hypothesis test, assuming a normal distribution. The critical values at the 1%, 5%, and 10% levels are presented for reference; the test statistic values are far beyond these values – indicating stationarity.

Table 3 presents autocorrelation, partial autocorrelation, Q-Stat, and associated probabilities. Thus, autocorrelation and partial autocorrelation were calculated to determine the correlation between time series and its lags at different lags. In addition, Q-Stat was calculated to determine the departure from the null hypothesis of no autocorrelation. The Q-Stat and their probabilities were small for every lag despite the possibility of autocorrelation in the time series. Table 4 depicts

Table 3: Autocorrelation and partial autocorrelation.

| Lag | AC | PAC | Q-State | Prob. |
|---|---|---|---|---|
| 1 | 0.093 | 0.093 | 3526.414 | 0.000 |
| 2 | -0.027 | -0.036 | 3529.005 | 0.000 |
| 3 | -0.010 | -0.004 | 3529.349 | 0.000 |
| 4 | 0.009 | 0.009 | 3529.649 | 0.000 |

Table 4: Estimation of volatility on SGARCH, GJR-GARCH and E-GARCH.

| Variables | ARCH | | S-GARCH | | GJR-GARCH | | E-GARCH | |
|---|---|---|---|---|---|---|---|---|
| | Coef. | p-value | Coef. | $p-$value | Coef. | $p-$value | Coef. | $p-$value |
| Omega | 1.109 | 0.000 | 0.059 | 0.003 | 0.095 | 0.000 | 0.031 | 0.000 |
| Alpha | 0.222 | 0.000 | 0.108 | 0.000 | 0.012 | 0.000 | 0.235 | 0.000 |
| Beta1 | - | - | 0.891 | 0.000 | 0.836 | 0.000 | 0.667 | 0.177 |
| Gamma1 | - | - | - | - | 0.289 | 0.000 | -0.17 | 0.000 |

the estimated coefficients and relevant $p-$values for four volatility models, ARCH, S-GARCH, GJR-GARCH, and E-GARCH. They are well-established volatility forecasting models that find their application in financial and econometric analysis. Specifically, the estimated coefficient for ARCH's constant term, Omega, equals 1.109, suggesting a significantly positive influence on volatility forecasting ($p-$value = 0.000). The estimated coefficient for lagged squared residuals Alpha1 is 0.222, indicating a significantly positive effect on the predicting volatility $p-$value = 0.000. The estimated coefficient for the S-GARCH model constant term, which is Omega, equals 0.0587, similarly suggesting a significantly positive influence ($p-$value =0.003). In the S-GHARCH model, the estimated coefficient regarding the lagged squared residuals Alpha1 equals 0.108, and it positively influences the volatility prediction $p-$value = 0.000. The estimated coefficient for lagged conditional variance Beta1 equals 0.891, which also positively affects volatility prediction and has a $p-$value = 0.000.

The results are similar, but the different models would have different impacts on the other; the estimated coefficient for Omega, whose constant value is 0.095, shows that it is positive and significant at 000. The coefficient of Alpha, which is the lagged squared residual, is 0.012, and it is positive and significant at 000; this shows that they have a positive and significant relationship rho 1, which is the coefficient of the lagged conditional variance, is 0.836 showing that it has positive and significant lagged considering the major impacts on the issues at 000 and the coefficient of Gamma is 0.2889 positive and significant at 000. Meanwhile, In the E-GARCH the Omega: The estimated coefficient for Omega the constant is 0.031, which is positive and significant at 000; the Alpha, which is the coefficient of lagged and conditional variance of 0.235, shows a positive and significant relationship at 000. Moreover, the beta, the estimated coefficient of lagged conditional variance, is 0.667, and there is a positive relationship but significant at 0.177. The expression of the exponential lagged conditional variance is -0.170, and a negative relationship shows that it is substantial at 000.

Our study investigates volatility prediction by using the four-type model. As explained before, the coefficients and $p-$values provide information on the statistical significance of the effect of each variable and the magnitude of impact it has on predicting volatility. In the case of the ARCH model, Omega and Alpha1 were both statistically significant positive impacts. This suggests that past volatility and unconditional variance are essential in determining future fluctuations. Both were positive coefficients, implying that the volatility should rise when the two variables increase. In the case of S-GARCH, we find both statistically significant positive impacts. The three variables are Omega, Alpha1, and Beta1. This indicates that the unconditional variance of past volatility and the lagged conditional variance should determine future fluctuations. All three coefficients are positive, leading to a higher predicted volatility when they increase.

The results show that the constant term in the GJR-GARCH model lagged squared residuals, the lagged conditional variance, and the lagged harmful squared residuals have statistically significant impacts from volatility prediction. The positive coefficients of Omega, Alpha1, and Beta1 imply that these variables increase volatility. On the other hand, the negative sign of Gamma1 reveals that the negative has a dampening impact on future volatility. For the E-GARCH model, all constants and variables, namely the constant term, the lagged squared residuals, and the lagged conditional variance, have a statistically significant impact on the volatility prediction. This is consistent with the previous two models, indicating the critical role of unconditional variance, past volatility, and lagged conditional variance in predicting future volatility. The negative sign of the exponential lagged conditional variance suggests that the volatility is persistent in a high volatility state, resulting in future fluctuations in subsequent periods. Overall, the positive and negative signs of the GARCH model and the GARCH model reveal the roles and some of the aspects that influence volatility. Since the sign of the E-GARCH model and E-GARCH model, the statistically significant coefficient variables capture one of the critical aspects of volatility dynamics. Thus, this can be very important in asset pricing, risk management, and investment strategies.

### 3.1. Assessment of volatility prediction

This study evaluated and compared various volatility prediction models, including GARCH, GJR-GARCH, EGARCH, SVM-GARCH hybrid, and neural network models. According to the statistical metrics, such as Root Mean Square Error and Mean Absolute Error, we have proven that the SVM-GARCH hybrid with a linear kernel and the neural network models showed better accuracy in predicting volatility than other models. The SVM-GARCH and neural network did a good job determining the complex volatility patterns and predicting future values, which positively affects financial analysis, investment decisions, and risk management. Moreover, the study results indicate that the SVM-GARCH hybrid with a linear kernel and neural network models represents a new level of prediction accuracy and the ability to capture patterns compared to the traditional GARCH-type models.

Thus, it enhances volatility prediction and can be applied in various financial contexts.

We have comprehensively evaluated several volatility models' predictive performance when forecasting future volatility. The accuracy of the predictions made by a model was determined by using two standard metrics: Root Mean Square Error and Mean Absolute Error. The lower the value of these metrics, the better the prediction is. As indicated above, the following results were achieved by some of the models evaluated in this research: S-GARCH, GJR-GARCH, and E-GARCH models provided relatively close results. According to Table 5, if the models were S-GARCH, GJR-GARCH, and E-GARCH, the RMSE and MAE values were 0.0039 and 0.0030, 0.0046 and 0.0033, 0.0048 and 0.0034, respectively. Thus, all three models demonstrated approximately equivalent and high-quality volatility prediction, although there is still room for improvement. Conversely, the SVM-GARCH model

Table 5: The performance of GARCH and SVM-GARCH hybrid model.

| Model | RMSE | MAE |
|---|---|---|
| S-GARCH | 0.0039 | 0.0030 |
| GJR-GARCH | 0.0046 | 0.0033 |
| E-GARCH | 0.0048 | 0.0034 |
| SVM-GARCH (Linear Kernal) | 0.000731 | 0.000481 |
| SVM-GARCH (RBF Kernal) | 0.001644 | 0.001222 |
| Neural network | 0.000862 | 0.000631 |

with a linear kernel had the most beneficial performance metric. The RMSE and MAE metrics had the lowest scores of 0.000731 and 0.000481, meaning the average volatility predicted by the model is precise. The SVM-GARCH with a linear kernel has, therefore, the highest indicator that should make an accurate prediction. The neural network model also showed decent results, with RMSE 0.000862 and MAE 0.000631. Indeed, these indicators are practically flawless and perfect compared to the other models. Summing up the overall results, SVM-GARCH with a linear kernel and neural network models were the best in terms of RMSE and MAE (Figure 4). They are the two most beneficial models that could predict the most accurate value of the model. Although this seems significant, other factors such as computation, possibilities of application, and accuracy may also impact the choice of model. In conclusion, the overall result is that the SVM-GARCH model with a linear kernel, alongside the neural network model, provides the best performance in volatility forecasting compared to traditional GARCH-type models. These models predict more precise information and are valuable when necessary to obtain the correct volatility value.

## 4. Conclusion

Our study offers novel perspectives on the volatility of Islamic equity funds and its substantive implications for financial practitioners and policymakers. By assessing a range of volatility models – including ARCH, S-GARCH, GJR-GARCH, E-GARCH, our hybrid SVM-GARCH, and NN – we have successfully pinpointed critical volatility drivers while enhancing the predictive accuracy of volatility. Incorporating past volatility and unconditional variance, lagged conditional variance, squared residuals, and constant terms are essential for predicting the volatility of Islamic equity funds. Regarding each model, the SVM-GARCH hybrid model with a linear kernel and the NN model provided a better volatility forecast for the Islamic equity funds index. When compared with RMSE and MAE, both of these models beat GARCH. These results indicate that the NN and SVM-GARCH offer new avenues for capturing the complexity of volatility patterns and predicting volatility more accurately. Our research has several practical implications. Policymakers will

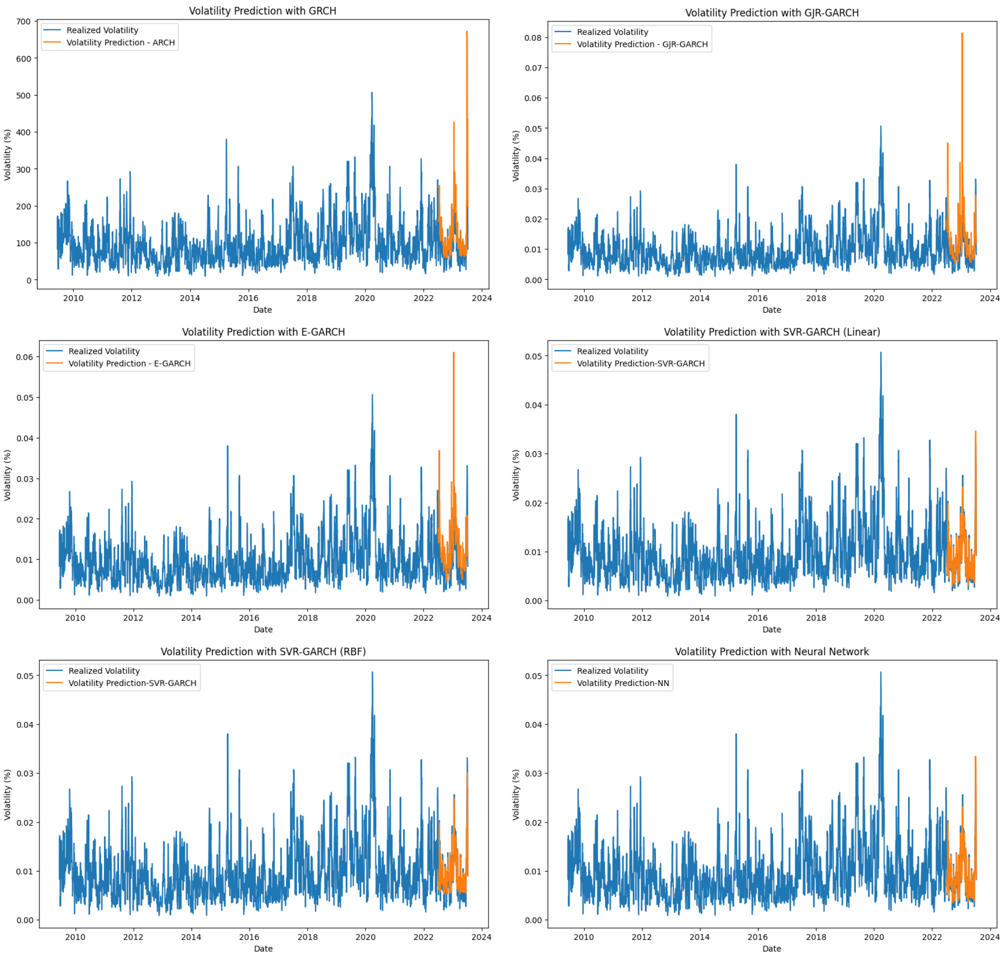


Figure 4: The actual and predicted volatilities.

benefit from the insights acquired from our volatility models, helping them make decisions on risk management, financial stability, and market policies. Knowing the factors influencing volatility will help policymakers develop appropriate policies that keep market fluctuations under reasonable control and maintain a well-functioning economic system. Similarly, financial practitioners can use our findings, incorporating the superior predictivity of our SVM-GARCH hybrid and neural network models into their portfolio management, risk assessments, and investment endeavors. It must be highlighted again that our research's methods and probing insights transgress Islamic equity funds alone. The volatility models scrutinized in our paper constitute a sound methodological toolkit to uncover and anticipate volatility throughout different market situations. Consequently, our research offers insights into Islamic equity funds' volatility characteristics and practical implications to policymakers and practitioners. It provides them with instruments to better understand their environment, make more informed decisions, manage risks more effectively, and navigate financial markets.

Further research could explore the volatility patterns of Islamic equity funds during major financial crises, such as the Global Financial Crisis and the COVID-19 pandemic, to identify any structural breaks or regime shifts and assess the performance of the GARCH-SVM hybrid model in capturing these changes. Conducting a comparative analysis between the volatility dynamics of Islamic and conventional equity funds could provide valuable insights into the risk profiles and diversification benefits of Islamic investments.

Additionally, integrating COVID-19 related factors, macroeconomic conditions, and regulatory changes into the hybrid model could enhance understanding of the complex relationships that influence the volatility patterns of Islamic equity funds. Expanding the application of the GARCH-SVM approach to other Islamic financial instruments, such as Sukuk and Islamic mutual funds, could further demonstrate the model's adaptability and contribute to the overall resilience and competitiveness of the Islamic finance ecosystem.

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